THRESHOLD RESPONSE MODELS APPLIED TO BEST-OF-N IN SWARM ROBOTICS

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Declaration of own work

I declare that the work in this MSc dissertation was carried out in accordance with the requirements of the University's Regulations and Code of Practice for Research Degree Programmes and that it has not been submitted for any other academic award. Except where indicated by specific references in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the dissertation are those of the author.

Sathvik Kadimisetty. 5th September, 2023.

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Abstract

Swarm robotics has emerged as a critical research area, particularly inspired by the collective behaviours and decentralized decision-making processes observed in social insects such as ants, and bees. This research aims to take inspiration from such natural social networks to offer a comprehensive exploration of threshold response models applied to the best-of-n problem in swarm robotics. The study builds upon existing theories and frameworks discussed in the literature, including natural swarm behaviours and decentralized control mechanisms, to offer a more balanced and effective approach to swarms that can choose from n>2 options. To validate these theoretical constructs, the research employs a rigorous simulation-based methodology. Various environmental conditions and constraints were considered to test the adaptability and resilience of the proposed models. The findings indicate significant improvements without compromising the quality of the decisions made by the swarm. In-depth discussions of these results reveal additional insights, such as the impact of threshold sensitivity on decision quality and the scalability challenges associated with an increasing number of options. While the study provides robust evidence in support of the proposed models, it also acknowledges limitations, primarily the reliance on simulation data. As such, future research directions include empirical validation using physical robot swarms and the fine-tuning of model parameters to adapt to specific environmental conditions. Overall, this dissertation serves as a contribution to the field of swarm robotics, offering a theoretically grounded, robust, and efficient model for decentralized decision-making in complex, dynamic settings.

Number of words in this dissertation: 8329 words.

Contents

1 Introduction

Nature offers a wealth of mechanisms honed over millennia for efficiency, resilience, and adaptability. Among these, the collective behaviours of social insects such as ants, bees, and termites are particularly noteworthy. Despite their individual simplicity, they undertake complex, synchronised tasks at the colony level, ranging from sophisticated nest construction to proficient foraging. These natural phenomena have been a source of inspiration for technological innovations, particularly in the field of swarm robotics for a long time.

Swarm robotics is an evolving domain where autonomous agents collaborate to tackle intricate challenges. At the core lies the concept of collective decision-making, which spans from binary decisions to multi-option and consensus decisions.

1. **Binary Decisions**: Visualize ants being attracted to two lights of varying brightness and see how they as a group work to choose the brightest one.

2. **Multi-option Decisions**: Envision collectives sifting through an array of options to determine the best course of action—a complex dance epitomizing the best-of-n problem.

3. **Consensus Decisions**: Gaze upwards to witness the fluid choreography of bird flocks—a harmonious representation of decisions achieved through consensus among individuals.

This research focuses on the best-of-n problem. This problem demands a collective decision from the swarm on the most suitable among 'n' available options. For instance, honeybee swarms, when faced with the challenge of relocating, must collectively select the most suitable nesting site from a set of available options. This decision-making process is not arbitrary; it is guided by a series of interactions and evaluations, where individual bees assess the quality of potential sites and communicate their findings to the swarm. This involves scout bees embarking on exploratory forays, meticulously assessing site attributes, and returning to communicate their findings. Despite its frequent occurrence, both in natural settings like honeybee swarms and in computational models, the best-of-n problem remains an area ripe for exploration.

Traditional computational models often employ intricate voter mechanisms and weighted strategies to arrive at an optimal decision. While effective, these methods are computationally demanding. This research adopts a different trajectory by exploring the elegance and computational thriftiness of threshold response models. Inspired by the simplicity observed in natural systems and social networks, these models operate based on individual and neighbour thresholds, offering a resource-efficient alternative for tackling the best-of-n problem.

This research aims to take inspiration from natural social networks such as an ant colony or a bee hive or even societal norms, to offer a comprehensive exploration of threshold response models applied to the best-of-n problem in swarm robotics. The research holds promise for significantly optimising problem-solving capabilities within the field, notably in heterogeneous swarms, by drastically reducing the computational and memory resources required.

In achieving this aim, the research will employ a multi-pronged methodology that includes the development of time-stepped multi-agent models in Python, and simulations to analyze model effectiveness. A significant part of this exploration will involve introducing noise and randomness into the models, thus aligning the theoretical framework with practical applications.

Through meticulous analysis, critical evaluation, and an extensive survey of existing literature, this research aspires to contribute a resource-efficient and robust approach to the best-of-n problem in swarm robotics. The ambition is not merely to uphold academic rigour but to potentially catalyse transformative advances in robotics, with far-reaching applications including precision agriculture and disaster response.

The subsequent sections of this thesis are organised as follows: Section 2 presents a literature review related to swarm robotics and decision-making processes, offering contextual grounding. Section 3 introduces our multi-agent models that utilise threshold-based decision-making criteria, presenting a novel approach to the best-of-n problem. Section 4 provides an in-depth analysis of the model results, exploring the impact of varying thresholds and the number of options. The thesis concludes with a discussion of the findings and avenues for future research.

In sum, this thesis seeks to occupy a unique niche at the intersection of biology, computation, and robotics. It aims not only to contribute a resource-efficient and robust approach to the best-of-n problem in swarm robotics but also to catalyze transformative advances in a range of applications, from precision agriculture to disaster response.

1.1 Aim

This research aims to investigate the viability of employing threshold response models in the context of the Best-of-n problem, to enhance the robustness of swarms operating in dynamic and noisy environments. Specifically, the research seeks to replicate the decentralised decision-making mechanisms present in natural social networks that use individual threshold responses as factors in collective decision-making.

1.2 Objectives

1. Develop simple time-stepped multi-agent models using threshold response and the best-of-n problem in Python.
2. Conduct simulations of all developed models and compare and analyze the results to identify more effective models.
3. Evaluate the performance of the developed models in terms of their ability to achieve consensus in the swarm and the scalability of the approach with larger swarms

1.3 Project Management

I was able to define the research aims, objectives, and methodology with the guidance of my supervisor. We made a detailed project plan that included key steps, what we needed to produce, and a timeline. Regular catch-ups with my supervisor helped keep everything on track and allowed us to make changes when needed. This good planning and regular advice were key to meeting all our deadlines.

We were also prepared for potential issues, like not having the right hardware, and unexpected results from our models (refer to C). Due to our preparations, we shifted our focus mainly to further analysing the models to get a more comprehensive understanding of our results. Open communication with my supervisor was crucial during these times, making sure the project stayed on course. This communication helped us in achieving the current results and the success of our project.

2 Literature Review

2.1 Collective Consensus Decision-Making

In the field of swarm robotics, the aspiration to achieve collective intelligence [1], [2], [3], [4] parallel to natural systems like ant colonies, avian flocks, or aquatic schools has become a cornerstone of research. This intelligence is predicated on the principle of "collective rationality," a concept that denotes decision-making processes as an emergent property of the swarm [5], rather than the isolated computation of individual agents. To delineate the complexity of collective decision-making, M. Schranz, et al.,[4] contribute a formalized framework and taxonomy for the swarm behaviours commonly encountered in robotics. This formalism assists in classifying and decomposing decision-making scenarios, thereby providing a structured and systematic approach to consensus problems. In tandem, Raoufi, et al.,[6] extend the scope of such decision-making to continuous, dynamic environments. Their methodology employs a two-phase algorithm consisting of exploration and exploitation stages. This adaptive framework allows robotic swarms to perform real-time evaluations of environmental variables, aligning closely with the dynamic decision-making observed in natural systems like fish schools that constantly reassess their spatial locations based on multiple ecological factors.

Of the many possible ways to observe collective decision making [7], "negative updating," as proposed by Lee, et al.,[8], [9] is an approach to materialize this. This mechanism utilizes evidential updating through pairwise comparisons among options, followed by propagation of opinions using the Product Operator. The algorithm not only expedites the convergence time but also enhances the swarm's collective ability to identify optimal decisions, analogous to ant colonies'[2], [10] efficient foraging techniques[11].

The intriguing question of how basic robotic units with limited computational power can execute such complex decision-making processes finds its answer in the principle of "self-organization."[12] This concept explored in-depth by Camazine, et al., elucidates that global complex behaviours can emerge from local interactions guided by simplistic rules. This emergent complexity is further systematized through the concept of "swarm engineering," as introduced by Brambilla, Ferrante, Birattari, and Dorigo[7]. Their methodology posits a set of design and analysis techniques that facilitate the development of robust and flexible swarm systems, thereby enhancing their collective decision-making efficacy.

Intriguingly, the concept of "tipping points," predominantly investigated in the realm of social sciences by researchers such as Nyborg, Anderies, Dannenberg, et al.[13], finds significant applicability in the context of robotic swarms. This theory states that minor perturbations in the behaviours or states of a select few agents can act as a catalyst for a system-wide transition, thus highly influencing collective decision-making. Similarly, work by Robinson et al.[14] asserts that a mere threshold is sufficient to account for complex collective decisions. They argue that the collective decision-making process becomes considerably more streamlined yet effective based solely on the individual thresholds of agents. This insight, inspired by the behavioural traits observed in social insects, serves as the foundation for our hypothesis.

In summary, collective consensus decision-making in swarm robotics emerges as a multi-disciplinary field, assimilating methodologies and principles from computational intelligence, formal logic, systems engineering, and social science. These systems, by virtue of their collective rationality, are poised to exceed the computational and operational limitations of individual agents, marking a significant advancement in the realm of artificial intelligence and robotics.

2.2 Best-of-n problem

This study centres on research addressing the consensus achievement problem, a complex issue that encapsulates a broad array of scenarios likely to be encountered by a robotic swarm. These scenarios range from selecting the most efficient path for transit to determining the optimal shape to form or identifying the ideal rendezvous location[7]. The intricate nature of solving the "Best-of-n Problem" in swarm robotics has given rise to a myriad of methodological approaches, each offering distinct contributions to the collective decision-making paradigm.

In seminal research conducted by Valentini et al.[15], a formal framework and taxonomy are introduced for the best-of-n problem. This framework categorises the problem based on the dual attributes of quality and cost associated with each available alternative. This formalisation affords a structured perspective for evaluating diverse strategies aimed at solving the best-of-n problem, thereby facilitating a comprehensive literature review and identification of extant research gaps.

The Best-of-n Problem mandates a robotic swarm to collectively determine which among the n available options serves as the most suitable choice to meet the swarm's immediate requirements[15], [16]. The term 'options' is context-dependent and varies according to the specific application at hand; for instance, it could refer to foraging patches or travel routes, where the options might be the number of available patches or the directions in which the swarm could move, respectively. Notably, the majority of existing research on the best-of-n problem is centred around binary decision-making scenarios, with limited focus on cases where n>2. Lee et al.[9] suggest that this limitation likely stems from the positive feedback approach's tendency to saturate option quality as n increases. A novel methodology has been proposed to address this limitation, demonstrating effective performance in scenarios involving more than two options, and has been further refined in subsequent studies[8].

Further advancing the field, the paper by Reina, Marshall, Trianni, and Bose[17] extends our understanding of best-of-n decision-making by focusing on the mathematical modelling of honeybee behaviours, considering scenarios with more than two options. This research introduces parameters that distinguish between rates of spontaneous and interaction-based transitions, thereby offering nuanced insights into swarm decision-making dynamics.

The concept of individual acceptance thresholds, while extensively explored in biological contexts, finds particular applicability in swarm robotics. Research by Masuda and colleagues[18] presents a computational model centred on the ant species Temnothorax albipennis. This model illustrates how ants with varied acceptance thresholds for nest quality contribute to robust and adaptive collective decision-making, enriching the methodology for addressing best-of-n problems in robotic swarms.

Minimalistic approaches are also gaining scholarly attention [16], [19], as evidenced by the work of Agrawal, Baliyarasimhuni, and Reina. They propose algorithms employing heterogeneous response thresholds to differentiate option qualities. Within the swarm, each robot assesses an option and issues a binary response—either acceptance or rejection—based on whether the option's quality surpasses its individual threshold. The researchers provide equations to optimise these thresholds, enhancing the swarm's collective decision-making process.

In summary, the multifaceted landscape of research surrounding the best-of-n problem in swarm robotics incorporates algorithmic innovations, formal taxonomies, biological insights, minimalistic design philosophies, and even topology considerations. Each of these research angles enriches our collective understanding of this complex problem, showcasing the diversity and depth of methodologies available for advancing swarm robotics.

2.3 Threshold Response Models

The research landscape surrounding "Threshold Response Models" forms a vibrant tapestry, seamlessly integrating insights from natural systems [7], [10], [14], [17], [18], [20], [21], [22] social sciences [23], and robotics[14]. For example, the paper by Jones et al. delves into the task allocation mechanisms within honeybee colonies, introducing the "response threshold model"[22]. In this model, individual worker bees possess distinct thresholds for responding to various stimuli, thereby enabling specialised tasks to emerge as emergent properties. Intriguingly, the paper links this mechanism to thermoregulation, demonstrating that genetic diversity among worker bees contributes to stable nest temperatures. Such findings could serve as valuable blueprints for task allocation in robotic swarms, particularly in contexts requiring adaptability to environmental fluctuations, which we hope to achieve in our model.

Venturing into the social sciences, Valente's seminal work on "Social Networks in the Diffusion of Innovations" [23] offers a unique nuance to threshold modelling. The paper explores "threshold models" within social networks, suggesting that an individual's propensity to adopt an innovation is influenced by their social thresholds. Although not explicitly focused on swarm dynamics, the principles elucidated in this paper have potential applications for understanding how individual agents within a swarm might be influenced by their interconnectedness, thereby enhancing collective decision-making.

Robinson et al.[14] contribute a captivating perspective through their research on house-hunting ants. Employing a "simple threshold rule," the paper posits that ants utilise internal thresholds to assess potential nest quality. A Markov-chain model within the paper reveals that this simple rule suffices for complex collective decision-making, challenging the necessity for more intricate mechanisms such as quality comparisons.

Building upon this foundational concept, Yamamoto et al.[16] introduce a mechanism for collective rationality in decision-making. They argue that agents with variable "yes/no" thresholds can optimise collective decisions through a majority-making mechanism. This insight holds particular relevance for swarm robotics, where the dual imperatives of simplicity and effectiveness often guide algorithmic design. Similarly, Agrawal et al.[19] note that the system's simplicity does not preclude accurate and rational collective behaviour, attributing this to the heterogeneity of individual thresholds within the swarm.

The research by Talamali et al.[24] examines the impact of communication limitations on swarm adaptability. By applying these findings to threshold models, the paper opens new avenues for optimising adaptability in swarm robotics. This research casts fresh light on the topic, suggesting that restricted communication, akin to that observed in social insects, can enhance efficiency within robotic swarms.

In summary, the multidisciplinary body of research on threshold response models offers a rich foundation for advancing our understanding of collective decision-making in diverse systems, ranging from natural colonies to robotic swarms. Each study, with its unique focus and methodology, enriches the collective discourse, illuminating how simple, localised rules can manifest in complex, adaptive behaviours.

2.4 Opinion Dynamics in Social Networks

The field of Opinion Dynamics is rich and multifaceted, drawing upon a diverse array of disciplines including physics, sociology, and marketing[25]. A seminal work by Castellano et al.,[26] utilises statistical physics to explore various models, such as the Voter and Majority Rule models. While broad in scope, the paper's treatment of opinion dynamics is replete with insights that are highly applicable to swarm robotics, offering a framework for individual robots to adapt their behaviours based on local rules. This notion aligns well with the work of Ehrlich and Levin on "The Evolution of Norms"[27], where a "threshold" voter model is introduced. In this model, individuals are influenced by their neighbours and modify their opinions upon crossing a specified neighbourhood threshold. Such threshold-based approaches introduce a compelling layer of complexity to swarm robotics, suggesting mechanisms for individual robots to adapt based on local interactions.

Shifting the lens slightly, H. Peyton Young's research on "The Evolution of Social Norms"[28] presents a robust framework for comprehending how social norms—essentially collective opinions—evolve and stabilise within societies. Concepts such as "tipping points" [13]and "punctuated equilibrium" offer intriguing parallels for swarm robotics. These ideas indicate that robotic swarms could benefit from abrupt, collective behavioural shifts, facilitating rapid adaptability to dynamic environments. Such concepts can be integrated into our model as dynamic thresholds to enhance its adaptability. Complementing this, the paper by Nyborg et al., "Social Norms as Solutions," [13] reinforces the idea of tipping points and explores the potential for external policies to induce these critical shifts, thus adding a nuanced layer to the debate surrounding autonomy versus control in robotics.

Further enriching the discourse is Valente's research on "Social Networks in the Diffusion of Innovations," [23] which introduces "threshold models" within social networks. This paper posits that individual adoption thresholds can be influenced by one's social network, a principle that could serve as a cornerstone in the design and programming of swarm robotic systems. This is echoed in Luo et al.'s paper[29], which investigates the coevolutionary dynamics of knowledge and network structures, providing a pathway for robotic swarms to simultaneously optimise task allocation strategies and communication networks.

In summary, the interdisciplinary landscape of research on opinion dynamics offers invaluable principles that could be adapted for swarm robotics. From individual thresholds to social norms, each paper contributes a unique lens through which to understand how localised interactions can result in complex, adaptive behaviours. These insights could prove pivotal in the design of more effective and adaptable swarm robotic systems, with broad applications ranging from environmental monitoring to search and rescue missions.

2.5 Summary

This literature offers a sweeping overview of multidisciplinary research that informs and enriches the field of swarm robotics, particularly focusing on decision-making paradigms. The review delves into four main areas: Collective Consensus Decision-Making, the Best-of-n Problem, Threshold Response Models, and Opinion Dynamics in Social Networks.

Notable methodologies like "negative updating" and formal taxonomies offer algorithmic and structured approaches to collective decision-making. These are further enriched by biological insights, such as individual acceptance thresholds, which contribute to a more robust and adaptable collective process. The principle of self-organization and the phenomenon of tipping points emerge as cross-disciplinary themes, highlighting how simple, localized rules or minor behavioural changes can lead to complex and adaptive collective behaviours. The concept of individual thresholds, central to both opinion dynamics in social networks and threshold response models, offers a unifying thread. It suggests that the properties of individual units, whether they are social agents or robots, can have a magnified impact when considered as part of a network or swarm. Overall, this research of ours converges on the idea that simple, threshold-based rules and localized interactions can give rise to complex, collective decision-making capabilities, a notion that holds significant promise for advancing swarm robotics and solving the best-of-n problem.

3 Research Methodology

3.1 Initialization

Swarm robotics draws its inspiration from the collective behaviour of social insects, where individual agents adhere to simple local rules, culminating in complex, emergent global behaviours. The *best-of-n* problem is a classic decision-making problem in this domain. Traditionally, most research has been limited to scenarios with n $<$ 3, but broadening this to n $=$ 10 provides a more comprehensive exploration of the decision space, simulating environments with increased complexity.

In traditional models, agents communicate with nearby peers to garner opinions. The decision-making process is then influenced by the aggregated data and they use various means to compute that data and decide on an option. Our model introduces a novel approach, leveraging a 'Threshold'-based mechanism to process this data.

3.1.1 Network Generation

Central to our study is the use of small-world networks for simulating agent interactions. This model has garnered acclaim for its applicability across both biological and social systems, offering an optimal balance between local specialisation and global integration. To this end, we utilise Python's NetworkX library, which affords the flexibility to model various network configurations. For our specific research needs, we deploy a small-world network, the parameters of which include the number of agents (nodes), their respective neighbours (k), and the probability of rewiring (p). You can refer to the code here [30] and the general working of the code is explained here in Appendix A.

Figure 3.1 offers a visual representation of the generated network, showcasing potential agent communication pathways. Such networks are pivotal in understanding how information spreads and the consensus is reached within a swarm.

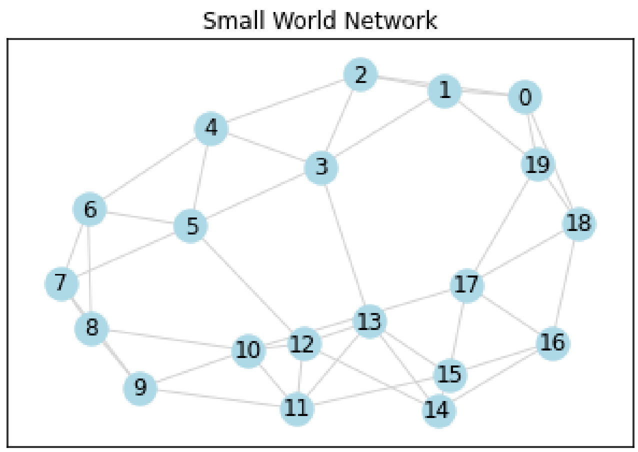


Figure 3.1: A Newmann-Watts graph with 20 nodes and 4 neighbours each, exemplifying node interconnectivity in our model.

3.1.2 Option Sampling

An essential aspect of our model is the options available for agents to sample. Each option is characterized by a quality metric q, and to ensure that the computation doesn't become complex, these qualities are equidistributed between 0 and 1. When agents sample an option, their perceived value X_n follows a normal distribution, introducing a realistic variability in their perception. Quality of options $(q) = [q_1, q_2, q_3, q_4, ..., q_{10}]$

Each agent would sample an option and the value X_n they sense would be in the range of a normal distribution curve with the mean as the quality q_n assigned to the location and a standard deviation σ. Incorporating a dynamic threshold, derived from the sensed quality X_n, introduces a layer of adaptability to our model. [30]

$$X_n \sim \mathcal{N}(q_n, \sigma) \tag{3.1}$$

This is a significant departure from traditional fixed thresholds, making our approach more resilient to environmental changes.

3.1.3 Threshold Functions

In our simulation, we are conducting a network-based decision-making process where each agent has a threshold λ value. However, since the options under consideration only differ in their quality value, we have decided to make the threshold a function of the quality sensed for each option. This value is then used as the threshold λ for any further communication and computation.

$$\lambda_n = f(X_n) \tag{3.2}$$

We have considered a few functions to generate the thresholds from the quality values. They are as follows:

$$\lambda_n = X_n \tag{3.3}$$

$$\lambda_n = 1/(1 + e^{(-X_n)}) \tag{3.4}$$

$$\lambda_n = min(X_n, a) \tag{3.5}$$

Here in 3.5, a is the highest threshold an agent can have. Further are in B.

3.2 Communication

Communication is integral to swarm robotics, and our threshold response mechanisms make full use of this. Agents exchange opinions with their neighbours and dynamically adjust their thresholds based on equation 3.2

3.2.1 Initial Information Exchange

The network is initialized by randomly sampling options, and the quality of each option is drawn from a normal distribution with mean q_n and standard deviation σ_n. The agents then determine their threshold based on equation 3.2 [19]. Unless otherwise specified, the threshold and quality function used in sampling is given by equation 3.3.[30]

$$X_n \sim \mathcal{N}(q_n, \sigma)$$

3.2.2 Peer Communication and Opinion Collection

Once initialized, agents enter into a more structured communication phase with their local networks 3.1. They not only broadcast their own opinions but also collect the opinions of their neighbours. This two-way information exchange is crucial for forming local majorities and facilitating collective decision-making.

3.2.3 Group Ratio and Local Majorities

If a group of agents broadcasts that the option they chose is the best, and their group ratio is the highest among all groups in that neighbourhood of the agent, they become the majority.[27]

$$\text{Group ratio (G)} = \frac{\text{number of neighbours in the group of Majority}}{\text{number of neighbours}}$$

If a majority emerges, surpassing an agent's threshold, the agent revises its opinion. This process iterates for every agent until a global consensus or stagnation is reached. Success is evaluated based on the alignment of the majority decision with the highest-quality option.

3.2.4 Dynamic Threshold Adjustment

An essential feature of our model is the dynamic adjustment of an agent's internal threshold (λ). If a local majority's opinion surpasses an agent's current threshold, the agent recalibrates its threshold according to equation 3.2 and updates its opinion [25], [13]. This dynamic adjustment provides the model with the flexibility to adapt to evolving circumstances and is a key differentiator from static threshold models.

3.2.5 Feedback Loops in Communication

Our model also incorporates feedback loops to allow for the possibility of agents revising their initial opinions. Once an agent has updated its choice, it communicates this new decision back to its local network. This iterative feedback process serves to strengthen local majorities and expedite the convergence to a global consensus or a stagnation point.

3.2.6 Impact of Network Topology

The structure of the small-world network plays a crucial role in communication. Agents in highly connected regions are likely to reach consensus more quickly but may also be more susceptible to 'groupthink,' whereby the swarm prematurely converges on a suboptimal choice. Conversely, agents in sparsely connected regions may take longer to reach a decision but have a greater chance of exploring multiple options[24]. Some of the other factors that come into play are the number of options to choose from, noise (σ), thresholds of the individuals, rewiring probability of our network and also the time we allow our agents to communicate.

By incorporating these additional features—dynamic threshold adjustments, and feedback loops our model aims for a comprehensive understanding of the communication processes that underlie swarm decision-making in the *best-of-n* problem.

3.3 Re-Sampling Strategy

Up until now, agents were allowed to sample options only once, during initialization. However, we hypothesize that allowing agents to re-sample the options, especially those endorsed by the majority could optimize the decision-making process. This re-sampling would serve as a validation step, ensuring that agents are more confident in their choices.[31]

By integrating these multiple facets—dynamic thresholds, group ratios, and re-sampling strategies—our model aims for a nuanced understanding of swarm decision-making. Further analyses will explore the interplay between these parameters to elucidate their collective impact on the *best-of-n* problem.[30]

3.3.1 Theoretical Framework

Our re-sampling strategy postulates that allowing agents to re-sample a previously endorsed option could add a layer of robustness to the decision-making process. This is premised on the idea that an agent's initial sampling may not always be representative of the true quality of an option due to noise, perceptual errors, or other random factors.

3.3.2 Re-Sampling Mechanism

Upon reaching a local consensus where the majority's choice surpasses an agent's threshold, the agent is programmed to re-sample the option endorsed by the majority. This re-sampling serves as a self-validation mechanism[26]. The re-sampled value, X'_n, is again obtained from a normal distribution centred around the option's quality:

$$X'_n \sim \mathcal{N}(q_n, \sigma)$$

The agent then recalculates its dynamic threshold λ_n^1 using the new sample:

$$\lambda'_n = f(X'_n)$$

3.3.3 Iterative Re-sampling

The re-sampling is not a one-time event but can occur iteratively, depending on the evolving dynamics of the swarm. Each time a new local majority emerges that surpasses the agent's current threshold, the agent may undergo another round of re-sampling and threshold adjustment.

3.3.4 Impact on Decision Quality

We hypothesize that this re-sampling strategy will refine the swarm's collective decision-making by allowing for more accurate assessments of each option. Furthermore, it may reduce the incidence of poor decisions that could otherwise arise from initial sampling errors or noise.

3.4 Experimental Validation

To validate the efficacy of the model, we will conduct simulations and compare the quality of decisions reached. Metrics for comparison will include:

1. Average Quality: The overall average quality of the network would help us see the change in opinion dynamics.

2. Robustness: How resilient is the decision-making process to variations in noise, network topology, and other parameters?

3. Alignment with Highest Quality Option: How often does the swarm's decision align with the highest quality option?

Through a meticulous investigation of the interplay between various parameters within the dynamics of swarm decision-making, our study aims to illuminate both the potential benefits and limitations with respect to the *best-of-n* problem.

4 Results

Our results are derived from an extensive range of simulations, each executed with meticulous attention to varying parameters. These parameters include the number of neighbours each agent interacts with, the level of noise during sampling, the distinct threshold functions applied, and the available options for agents to select. The below description explains how we have varied the parameters and arrived at our present results.

- **Working of the model**: A single run of our model is composed of several steps. Each agent initially chooses an option at random and samples quality based on a normal distribution, as delineated in Section 3.1. The threshold is then set according to Equation 3.2. Subsequently, agents engage in intra-network communication to form a majority opinion, which informs their final decisions. This whole process of communication is considered on a single time step. Each run comprises 200 time steps, allowing agents ample opportunity to converge on stable decisions. For each parameter set, we conducted 100 such runs, and the reported metrics represent the averages, thereby minimising edge cases and anomalies. So, for every value on the plots, we have completely ensured to get the best results that we can for that set of specific parameters.

The following metrics serve for comparative analysis between different runs and models explored in the subsequent sections. Although briefly introduced earlier, a more elaborate description is presented.

1. **Average Quality**: Average Quality or the Average Decision Quality is a measure of the quality of the swarm in its entirety. Calculated at each time step for all agents, the values presented subsequently represent the mean outcome of the final time step across a specific series of 'multi-runs' for a given parameter set. Each 'multi-run' consists of 100 runs unless otherwise specified.

2. **Alignment with the Best option**: Alignment is calculated for every 'multi-run'. At the end of every run, the predominantly selected option among the agents is identified. If this aligns with the best option, the run is scored as 1; otherwise, it receives a score of 0. Now, at the end of every 'multi-run', we see the percentage of runs that actually chose the best option majorly. This serves as an indicator of the model's alignment towards the best option within the context of specific parameters.

It should be noted that the aforementioned definition of 'run' pertains to the 'normal' method of operation and albeit similar with a small change, in the re-sampling strategy, an agent resamples the majority-chosen option within its neighbourhood only if its threshold is lower than the group ratio. This approach facilitates more accurate quality assessments and dynamic threshold adjustments.

4.1 Minimal communication enhances swarm performance

Contrary to the prevailing assumption that high levels of connectivity inherently benefit swarm robotic decision-making, our empirical findings suggest otherwise [24]. We observed that a systematic increase in inter-agent connectivity led to a statistical reduction in the overall average quality of the network.

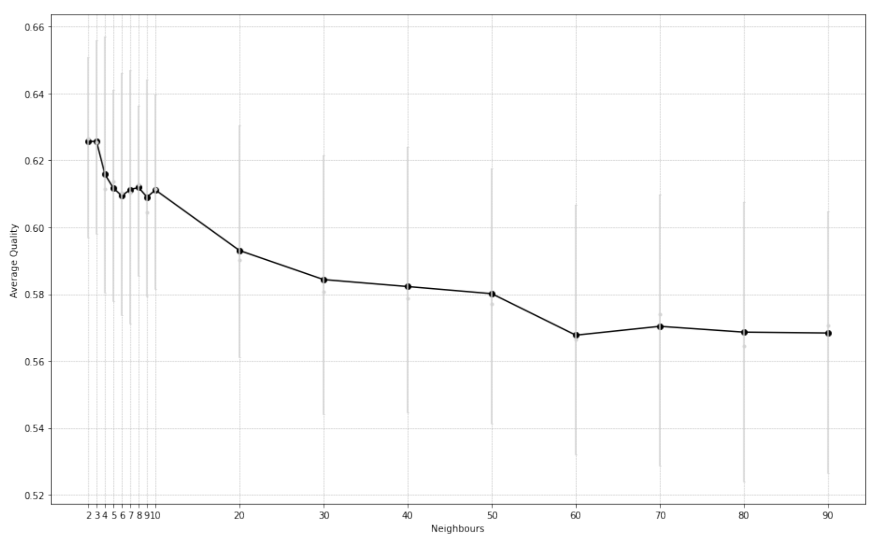


Figure 4.1: Inverse correlation between network connectivity and average decision quality.

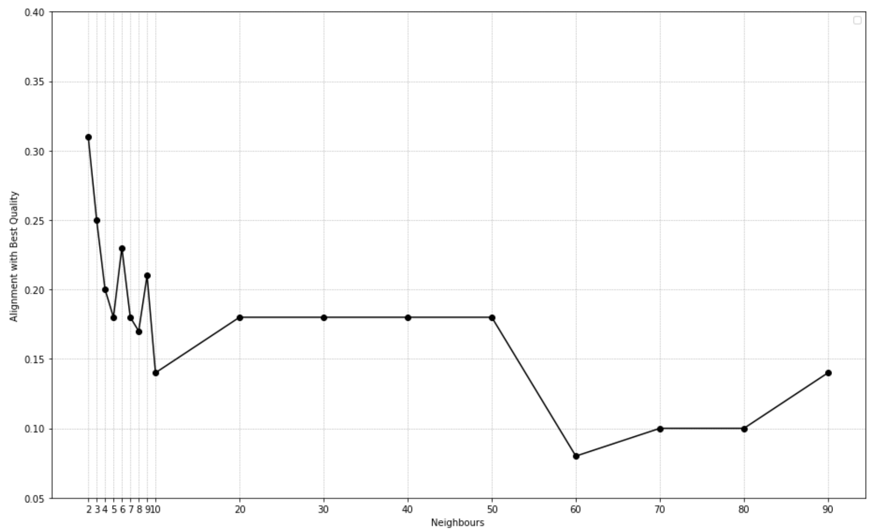


Figure 4.2: Alignment with the highest quality option across varying levels of connectivity

This might be due to the Mitigation of Information Redundancy, Limited connectivity reduces the chances of redundant information circulating within the swarm, thereby streamlining the decision-making process. This leads to more autonomous and less correlated decision-making processes among individual agents.

From fig.4.1, we observe that although the average quality seems to fluctuate at lower connectivity levels, the highest could be found for 3 neighbours and though there is a drop after, these variations are minimal when scaled appropriately. A more pronounced decline in average quality becomes evident as the agent-to-neighbour ratio surpasses 10. Although the decline isn't precipitous, it is gradual, indicating a trend where agents increasingly diverge from optimal solutions, meaning the agents are not converging towards the best option in terms of quality but their own self-assumption of the best option. We can also observe that there is a lot of noise in our results at higher levels of connectivity.

As illustrated in Fig. 4.2, the instances of alignment with the highest-quality option are generally rare but notably more frequent at lower levels of connectivity. This suggests that a higher average quality correlates with a higher probability of collective decisions aligning with theoretically optimal options. Our results of average quality and the alignment to the best option support each other claims.

4.2 Quality Augmentation through Iterative Re-Sampling

The concept of iterative refinement is a well-established paradigm in the realm of algorithmic optimization[31]. Intriguingly, our findings indicate that this paradigm is transferrable to the domain of swarm robotics, specifically when agents have the capability to re-sample previously assessed options or decision points. This mechanism introduces a multitude of benefits to the decision-making process, which can be enumerated as follows:

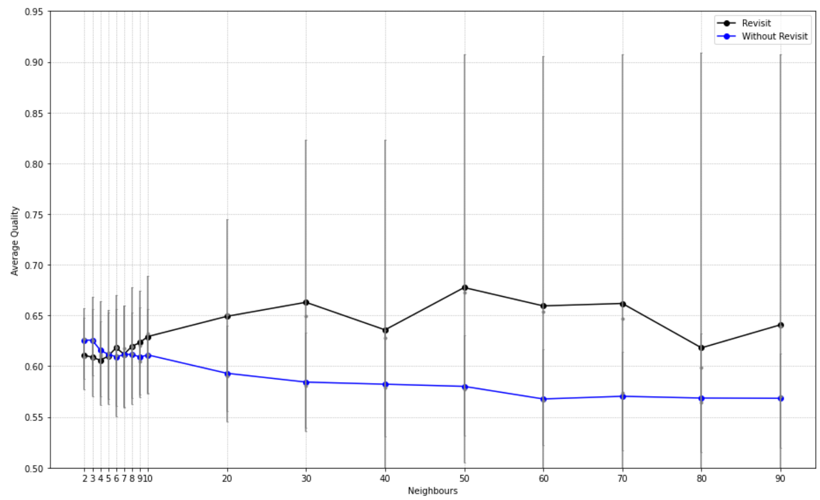
1. Dynamic Update of Decision Matrices: The act of re-sampling or re-visitation allows for the dynamic updating of decision matrices. This allows individual agents, and consequently, the swarm, to adapt to evolving environmental variables and newly acquired information.

2. Convergence towards Optimality: Similar to the mechanism of stochastic gradient descent in optimization algorithms, iterative re-visitation guides the swarm towards increasingly optimal solutions over time.

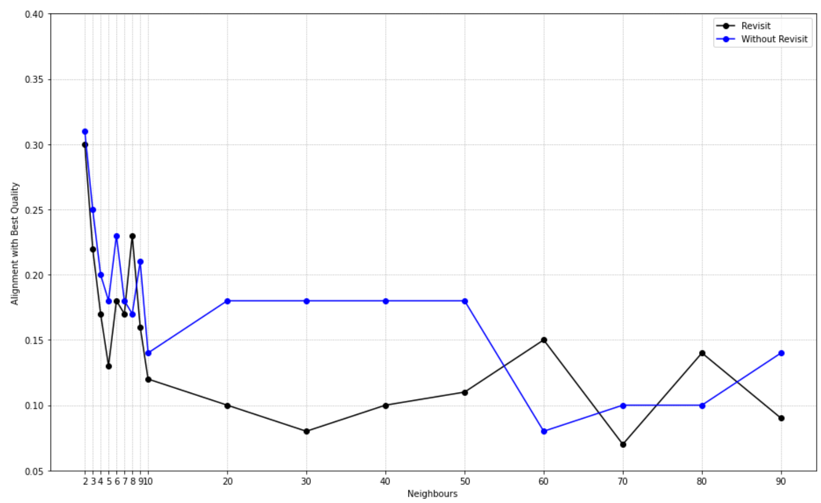
To validate the efficacy of this iterative strategy, we compared the decision quality metrics of swarms with and without re-visitation capabilities. As illustrated in Figure 4.3a, the data reveals a noticeable elevation in overall average decision quality for swarms that employed iterative re-visitation. While it is noteworthy that the highest quality does not always pertain to the re-visiting method, the consistently elevated average quality across diverse network connectivities underscores its utility.

Moreover, the alignment with the highest-quality options, as depicted in Figure 4.3b, further bolsters our hypothesis. These findings collectively demonstrate that the re-visitation strategy significantly augments the decision-making process, making it more robust and adaptive.

By introducing the concept of iterative re-visitation into swarm robotics, we are essentially imbuing the system with a mechanism for continuous learning and adaptability. This strategy proves particularly useful in complex, dynamic terrains, thereby amplifying the swarm's overall effectiveness and decision-making acumen.



(a) Comparison of average decision quality across varying levels of connectivity



(b) Comparison of Alignment with the highest quality option across varying levels of connectivity

Figure 4.3: Black represents With and Blue represents Without iterative re-visitation

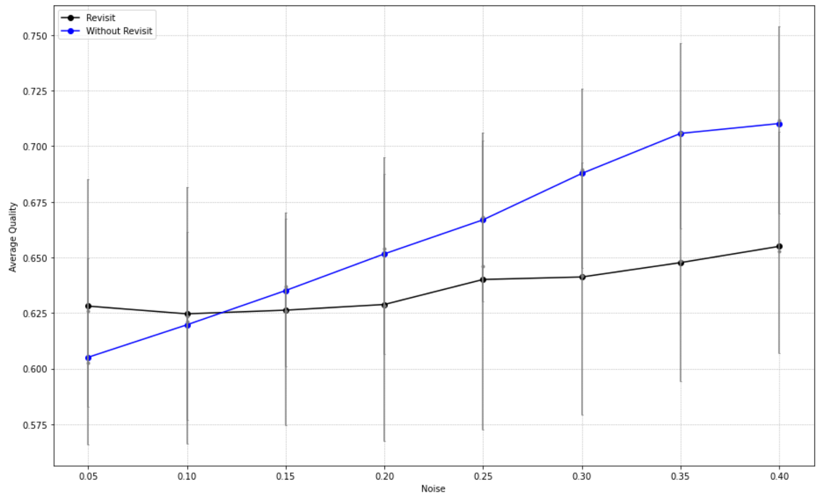
4.3 Impact of Noise Levels

Our analysis presents some noteworthy findings:

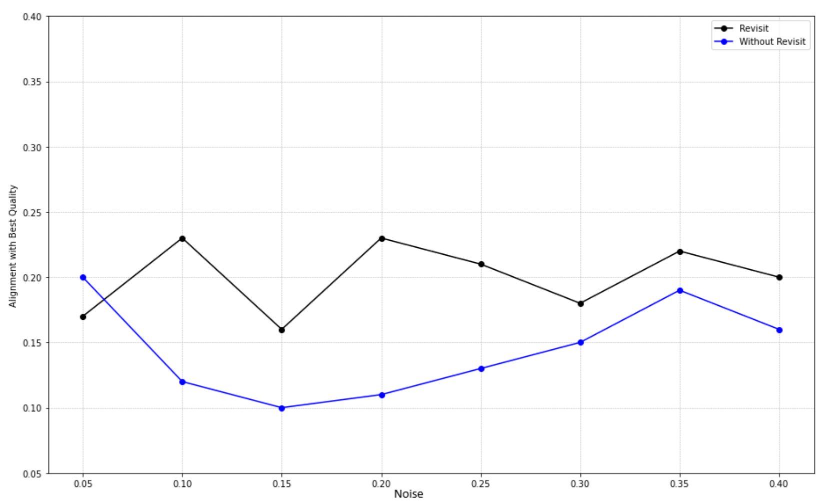
1. Robustness Against Noise: Intriguingly, our data indicates that noise levels don't significantly impede decision quality when robots are allowed to revisit previous options. This robustness against noise is noteworthy and speaks volumes about the model's

reliability. This is clearly depicted in Figures 4.4a and 4.4b.

2. Stable Decision-making Under Varied Conditions: The swarm showed admirable stability across different noise levels. Whilst there were occasional deviations from optimal decision-making, the swarm generally managed to recalibrate its choices effectively.



(a) Comparison of average decision quality across varying levels of noise



(b) Comparison of Alignment with the highest quality option across varying levels of noise

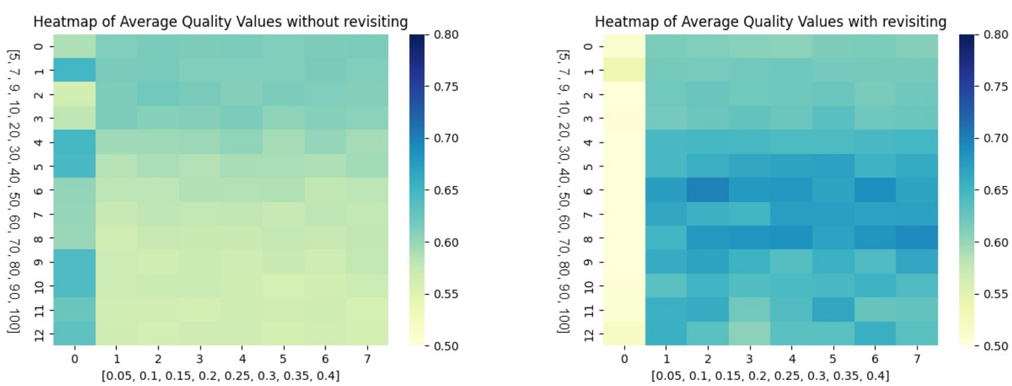
Figure 4.4: Black represents With and Blue represents Without iterative re-visitation

3. Value of Iterative Re-visitation: The act of revisiting past decisions led to improved

adaptability. When faced with increased noise, robots with this feature outperformed those without.

Interestingly, our data also reveals a paradoxical increase in the average quality of decisions in models where re-visitation is not permitted as noise levels escalate. One plausible explanation for this phenomenon could possibly be attributed to initial noise levels providing a form of random sampling, which may lead to serendipitous high-quality choices. In high-noise scenarios, the inherent variability could lead to a serendipitous alignment with higher-quality options. Once these choices are propagated within the network, they can have a lifting effect on the group's overall decision-making quality.

It's crucial to note that while an increase in noise levels does correspond to a minor decrement in alignment with the highest-quality options, this decline is not severe. More intriguingly, this decrement exhibits a tendency for recovery as noise levels continue to increase. This pattern is consistent across both the iterative re-visitation and non-re-visitation models, further substantiating the robustness of our swarm robotic framework.

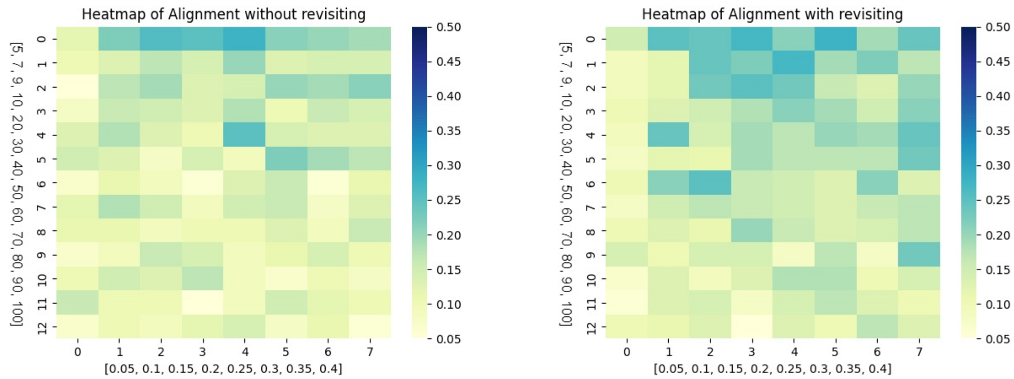


(a) Comparison of average decision quality across varying levels of noise and neighbours

(b) Comparison of average decision quality across varying levels of noise and neighbours

Figure 4.5: With visiting once and iterative revisiting respectively

In scenarios with elevated noise levels, the iterative re-visitation mechanism proves to be particularly advantageous. While we did observe a slight uptick in decision quality as noise levels escalated, the increase occurs only at extreme noise thresholds and, when we



(a) Comparison of Alignment with the highest quality option across varying levels of noise and neighbours

(b) Comparison of Alignment with the highest quality option across varying levels of noise and neighbours

Figure 4.6: With visiting once and iterative revisiting respectively

compare them against the model devoid of iterative re-visitation, is statistically negligible. This implies that the marginal gains in decision quality in high-noise conditions are effectively nullified by the re-visitation mechanism, highlighting its role as a stabilizing factor.

We also studied the impact of noise levels in conjunction with varying degrees of connectivity, as depicted in Figures 4.6a, 4.6b, 4.5a, and 4.5b. When we compare the noise results above, we can see a similar result in the lower noise level but as the noise increases also with the increase in the connectivity, even without re-sampling, our model is pretty robust, with minimal change in average quality. Remarkably, the data reaffirmed our prior findings on the benefits of lower connectivity, which were consistent even under varied noise conditions. This robustness across both noise and connectivity variables further emphasises the general resilience of our model.

So, in essence, our evidence proves that the model demonstrates resilience across varying degrees of noise. Furthermore, our findings also strongly advocate for the incorporation of iterative revisitation mechanisms in swarm robotics models as they consistently outperform models lacking re-sampling techniques, irrespective of noise levels and connectivity constraints. It not only leads to better decision outcomes but also significantly enhances the model's resilience to noisy and unstable conditions. This makes the model particularly well-suited for practical applications where environmental conditions can be unpredictable.

4.4 Performance Implications of Expanding Decisional Optionals

Increasing the number of decision points or options within a swarm robotic system introduces a unique set of challenges:

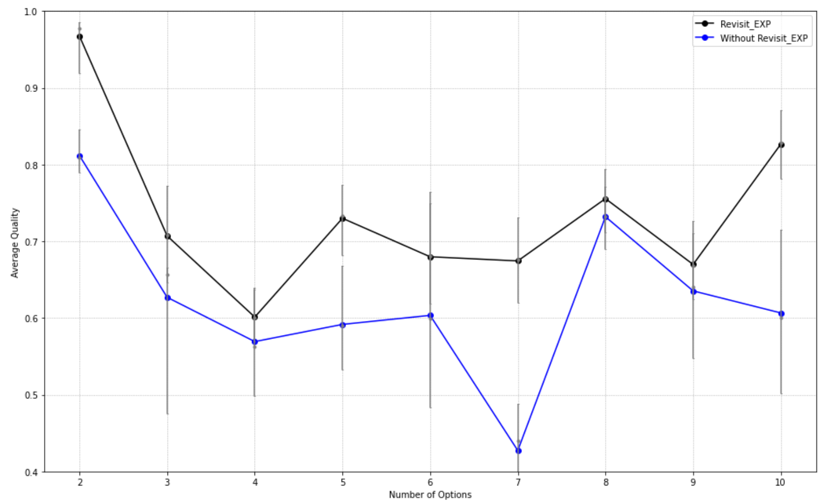
1. The escalation in the number of available options engenders an exponential augmentation of the decisional space. This phenomenon resonates with computational complexity issues in algorithmic contexts.

2. Processing Bandwidth: The swarm's processing capabilities are strained due to the newly introduced multi-dimensionality, as individual agents struggle with parsing an increasing array of choices.

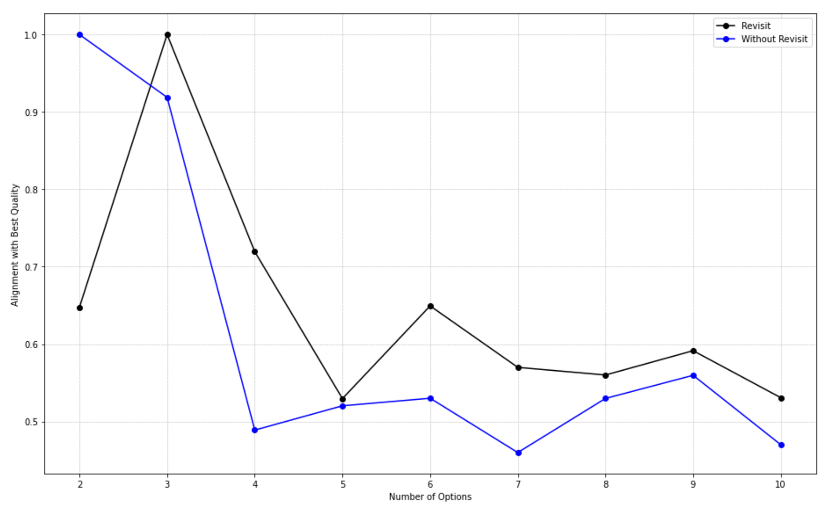
By examining Figures 4.7a and 4.7b, it becomes evident that the introduction of a larger pool of decisional options yields a noticeable impact. Specifically, the average decision quality exhibits a rather volatile pattern, though it generally shows a decline as the number of options increases from binary to a set of 10. This is indicative of the agents' struggle with the augmented complexity. More telling, however, is the decline in alignment with the best option, which serves as a more robust metric for evaluating decision-making efficacy.

Both models that were enabled with and without the re-visitation feature exhibited similar downward trends. This commonality underscores that the performance degradation is primarily attributable to the expansion in the available options, rather than the specific features.

To summarise, our findings accentuate the trade-offs involved in augmenting the number of decisional options within a swarm robotic system. While increasing the complexity of the decision-making environment might seem advantageous for its versatility, it comes at the expense of decision-making quality and computational efficiency. This is particularly noteworthy for real-world applications where the swarm might be deployed in environments that are resource-constrained or require rapid decision-making.



(a) Comparison of average decision quality across a varying number of options

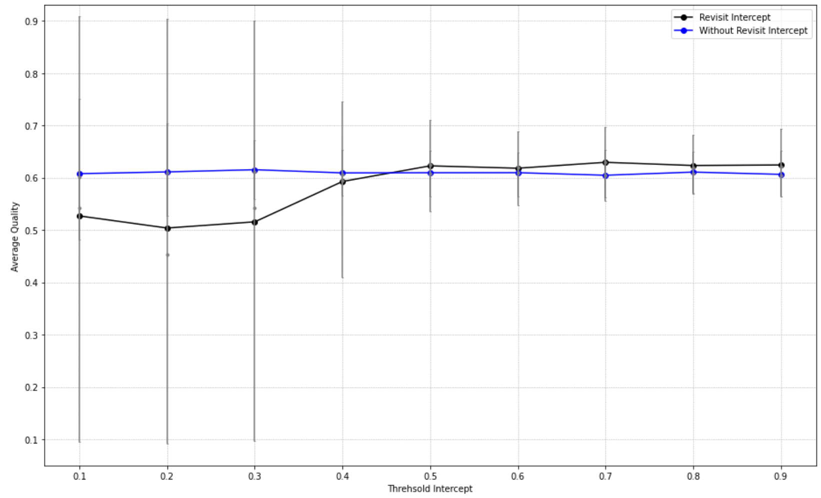


(b) Comparison of Alignment with the highest quality option across a varying number of options

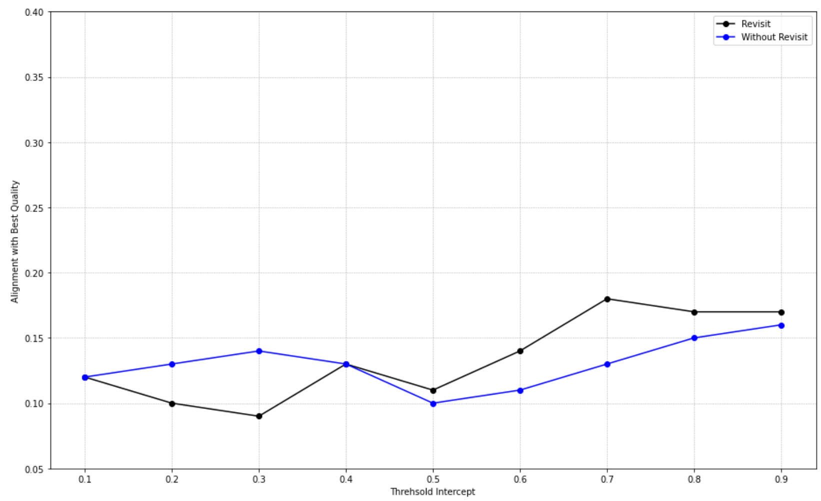
Figure 4.7: Erratic average decision quality and a decrease in alignment with best option

4.5 Threshold Manipulation and Invariance in Outcome

Our exploration into the efficiency of various threshold mechanisms yielded intriguing insights that challenge our assumptions. Specifically, the use of alternative threshold functions, beyond the linear function posited as the norm in Equation 3.3, did not yield any significant improvement in the average decision quality of the swarm. This observation suggests a couple of pivotal possibilities:



(a) Comparison of average decision quality



(b) Comparison of Alignment with the highest quality option

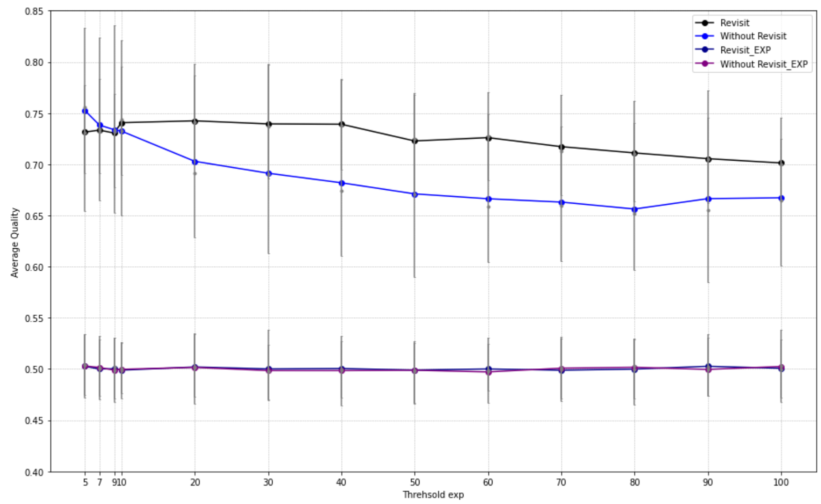
Figure 4.8: Comparative performance with variable intercepts in threshold function 3.5

1. Inherent Robustness of Threshold Mechanisms: One interpretation of these findings is that the swarm's decision-making process might be intrinsically robust to the types of threshold modifications we considered. This opens an avenue for further investigation into the robustness of threshold-based swarm decision-making.

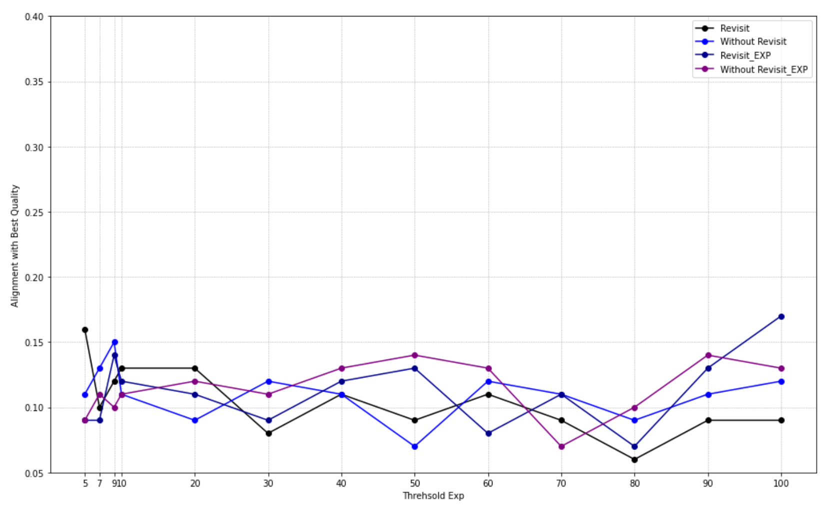
2. Overshadowing by Latent Variables: Alternatively, the absence of a noticeable impact from threshold adjustments could indicate the presence of latent variables or complex,

non-linear interactions that overshadow the effects of these modifications.

A meticulous examination of Figures 4.8a and 4.8b further corroborates our arguments. Here, we varied the intercept value in the threshold function outlined in Equation 3.5, and compared its impact on both average decision quality and alignment with the highest quality option. Remarkably, we found minimal to no deviation in performance metrics, irrespective of the intercept adjustments.



(a) Comparison of average decision quality



(b) Comparison of Alignment with the highest quality option

Figure 4.9: Performance comparison of linear (Equation 3.3) and exponential (Equation 3.4) threshold functions

Turning our attention to Figures 4.9a and 4.9b, we contrasted the linear function employed ubiquitously in our earlier experiments with an exponential function, as described in Section 3. The linear function unequivocally outperformed the exponential function in terms of average decision quality. Although the alignment metrics did not exhibit a recognisable pattern, it is worth noting that the highest alignment for the linear function occurred at low connectivity levels. This lends further credence to our hypothesis that reduced connectivity offers greater flexibility and, consequently, enhanced performance.

To sum up, the findings of this section add another layer of complexity to our understanding of the role of threshold mechanisms in swarm decision-making. While our results did not validate the utility of more complex threshold functions, they did reveal nuances that warrant further investigation, especially regarding the trade-offs involved in threshold manipulation within swarm robotic systems.

5 Discussion and Conclusion

In wrapping up this dissertation, I think it's crucial to revisit the initial points of inspiration. The decentralized decision-making processes observed in social insects like ants, and bees, as well as some societal norms. These have served as an inspiration in the field of swarm robotics for a long time.

We offer a comprehensive exploration of threshold response models applied to the best-of-n problem in swarm robotics with options of more than three. Our model provides a balanced, effective mechanism that doesn't compromise on essential attributes like the number of options, average quality and robustness. This unique integration aligns well with the observations made in natural swarms.

Moreover, our research validates the theoretical frameworks discussed, including collective consensus decision-making and decentralized control. The "best-of-n" models with threshold responses not only align with these theories but extend them by offering a more nuanced, adaptable mechanism for decision-making in swarm robotics.

Key achievements of our model include:

1. Provides empirical evidence to support theories of decentralized swarm behaviour.

2. Making novel technical and methodological contributions to the field.

3. It opens up avenues for practical applications in diverse fields, from search and rescue missions to environmental monitoring and beyond.

Our research yields several observations:

1. **Adaptability and Environmental Cues**: The feature of adaptability in our models emerges as a critical characteristic, one that is especially important when considering the swarm's capacity to adapt to real-time environmental fluctuations. Our simulations show that the swarm has the ability to adapt to real-time environmental changes, similar

to social insects. This adaptability is especially useful in unpredictable environments like disaster zones or rapidly changing weather conditions.

2. **Limited Connectivity**: Our results demonstrate the potential of limited connectivity in swarms. Agents with less number of neighbours have actually had a better performance than those with higher connectivity. This is due to the elimination of redundant data that is often collected with higher connectivity, which reduces the computational resources required for the simulation to take place.

3. **Robustness**: Another noteworthy observation is the robustness exhibited by the swarm under a variety of replicated environmental conditions(the variety of neighbours and the noise), similar to natural swarm systems. This level of robustness has profound implications for real-world applications, particularly in mission-critical scenarios where system failure is not an option. As this is a simulation, the parameters we can tune are limited and replicating the real-world scenarios in simulation software could yield clearer results.

4. **Threshold Sensitivity and Decision Quality**: Our simulations yielded intriguing insights into the role of threshold sensitivity in shaping the quality of decisions made by the swarm. Lower thresholds yield outcomes similar to those observed at higher thresholds across two of the varied threshold functions. While the linear function demonstrated comparatively superior results, suggesting its suitability for certain types of decision-making scenarios. This also opens up further research into the kind of relation the threshold functions have with the average decision quality of the swarm.

5. **Scalability and Complexity**: Our simulations show that the swarm can handle increasingly complex tasks, but the decision-making process becomes more elaborate as the complexity of decision nodes increases. This observation highlights the need for advanced algorithms that can scale efficiently with complexity.

Our research also contributes to the literature by demonstrating the resilience and adaptability of our models, which are more nuanced and effective than those limited to fewer than three options, aligning well with theories surrounding natural swarm behaviours and decentralised decision-making as outlined in our Literature Review section.

5.1 Limitations and Future Work

It is crucial to commence this section by candidly addressing the limitations of our research. While the model displayed robust alignment with the highest quality for n<6, our simulations revealed only modest improvements for options more than 6, even when supplementary resampling techniques were utilised. Specifically, the sampling methodologies employed could be refined—whether by increasing the number of sampling instances or altering the modality of option sampling. Moreover, the scope of our study was basically constrained by the use of simulated data and a relatively narrow range of decision-making scenarios.

Looking ahead, the research sets a fertile ground for future explorations. One intriguing line of enquiry can be the relation between the threshold functions and the overall quality of the model. We have tried the linear, exponential and other ways of relations B. This opens the door for more comprehensive studies focusing on this interesting relation that exists in nature, as evidenced by various research conducted with social insects.

Our simulation-based methodology while rigorous, merely serves as a precursor to what should ideally be validated through empirical studies using physical robot swarms in real-world settings. Future studies should concentrate on empirical testing and the refinement of threshold sensitivity parameters to better adapt to a range of environmental conditions and task requirements, thereby ensuring a universally applicable model.

The inherently versatile nature of swarm dynamics allows for myriad opportunities for future optimisation and fine-tuning. The swarm models can be adapted and extended in myriad ways, making them highly amenable to a range of applications from healthcare and disaster management to surveillance and beyond.

In conclusion, this dissertation stands as a seminal piece of research in the realm of swarm robotics. As automated systems grow more indispensable in modern life, the knowledge derived from our investigation shows promise for the development of highly adaptable and resilient robotic swarm systems. They hold the potential to usher in a new generation of highly adaptable and resilient robotic systems. This broadens the scope and impact of automation in various sectors.

5.2 Self Reflection

Embarking on this dissertation has been nothing short of transformative for me, both intellectually and personally. In the beginning, I had a basic understanding of swarm robotics, a subject that I always had an interest in. Under the skilled mentorship of Prof. Jonathan Lawry, that elemental awareness has blossomed into a well-rounded, nuanced grasp of the field.

The notion that simple creatures like ants could execute complex tasks in a decentralised manner has fascinated me deeply. This became the cornerstone of my research goals, leading me to investigate the computational models that could mimic such natural brilliance. The road to this moment was filled with hurdles, each serving as a lesson in perseverance and analytical problem-solving.

Prof. Jonathan Lawry's guidance has been pivotal. His approach encouraged me to delve deeper into the topic, rather than seeking straightforward solutions. This educational strategy has substantially improved my learning experience, transforming how I approach problems and encouraging me to think critically.

The complexity of implementing threshold models in our simulations presented a significant challenge when we had noise affecting a lot of our results but overcoming this obstacle was rewarding. While I acknowledge the study's limitations, such as the reliance on simulated data, these challenges serve as a stepping stone for future research opportunities.

Moreover, this project was not just a technical exercise but a lesson in academic rigour. Regular discussions with Prof. Lawry were much more than mere checkpoints; they were enlightening conversations that helped me evolve as a researcher. As I turn the page on this chapter, I look forward to the countless opportunities for further learning and exploration that lie ahead.

A Appendix

```
Class ThresholdResponse:
Method __init__(locations, agents, neighbours, noise, iterations,
thresh_fun, intercept, connectivity):
    number_of_locations <- locations
    options <- Empty List
    sd <- noise
    number_of_agents <- agents
    number_of_neighbours <- neighbours
    threshold_function <- thresh_fun
    intercept <- intercept
    connectivity <- connectivity
    number_of_iterations <- iterations
    iterations <- 0

Method generate_means():
    quality_samples <- List of predefined quality samples
    best_mean <- maximum of quality_samples
    sorted_list <- sort quality_samples in descending order

Method threshold(quality):
    return quality

Method threshold_a(quality, a):
    if quality < a:
        return quality
    else:
        return a
```

```

 Method threshold_2(quality):
     sigmoid <- sigmoid(10 * (quality - 0.5))
     return 0.5 * sigmoid + 0.25

 Method generate_network():
     Initialize a network graph
     For each agent in number_of_agents:
         Add agent to network graph
         Randomly connect agent to number_of_neighbours other agents

 Method sample_agent_qualities():
     For each agent in number_of_agents:
         Randomly sample quality from a predefined distribution
         Store the sampled quality for the agent in a data structure

 Method assign_thresholds():
     For each agent in number_of_agents:
         Retrieve the sampled quality for the agent
         Calculate threshold based on quality using a required function (e.g., sigmoid, linear)
         Store the calculated threshold for the agent in a data structure

 Method run_simulation():
     generate_network()
     sample_agent_qualities()
     assign_thresholds()
     While iterations < number_of_iterations:
         For each agent:
```

```
             Make initial choices based on some conditions
             Update choices based on neighbours choices and own threshold
         Update global statistics like best quality, second best quality, average qualities, etc.

         If converged:
             break
         iterations += 1
 Method evaluate_performance():
     Calculate metrics like adaptability, limited connectivity, robustness, etc.

 Method plot_results():
     Generate plots based on the simulation data

 Method save_results():
     Save simulation data and results to a file
```

B Appendix

B.1 Exploration of the Sigmoid Threshold Function

As our core research centres around threshold models that have demonstrated notable efficacy, we are curious about the effect the sigmoid function would have on our threshold. This mathematical function, often seen in the realm of neural networks, serves as a basis for our exploratory model. Although preliminary results didn't render it superior to the existing models, it adds an interesting dimension to our study.

B.1.1 Formulating the Modified Sigmoid Equation

Inspired by the foundational sigmoid function, we modified it to match the constraints of our project setup. We had to modify it in a way that it doesn't generate too low of a threshold or too high of a threshold. So, the equation that emerged from this adaptation is as follows:

$$\lambda_n = (0.5/(1 + e^{(-10(X_n - 0.5))})) + 0.25 \tag{B.1}$$

Here, the equation has been adjusted to fit within the (0,1) range on both the quality and threshold axes.

B.1.2 Evaluative Metrics

For a comprehensive understanding of this modified sigmoid function, we looked into two principal metrics. The findings are graphically presented below B.2:

B.1.3 Analytical Insights

Though our adapted sigmoid function didn't markedly enhance our model's output, its presence in our research serves multiple purposes. It extends the potential functional forms for

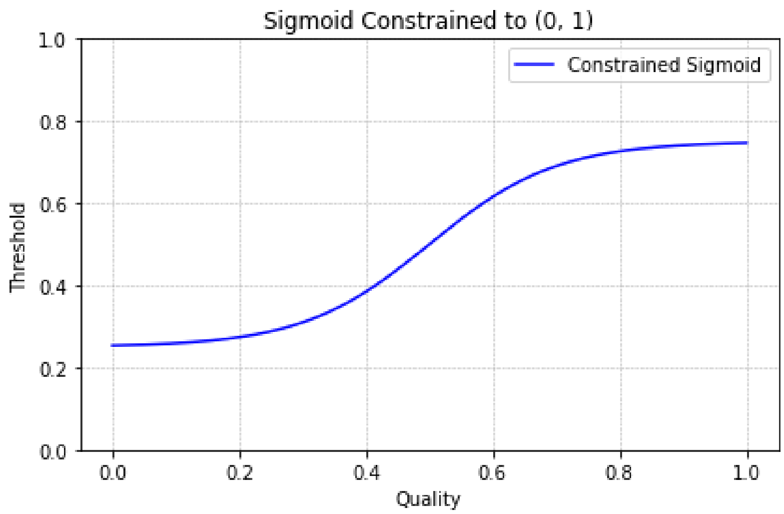
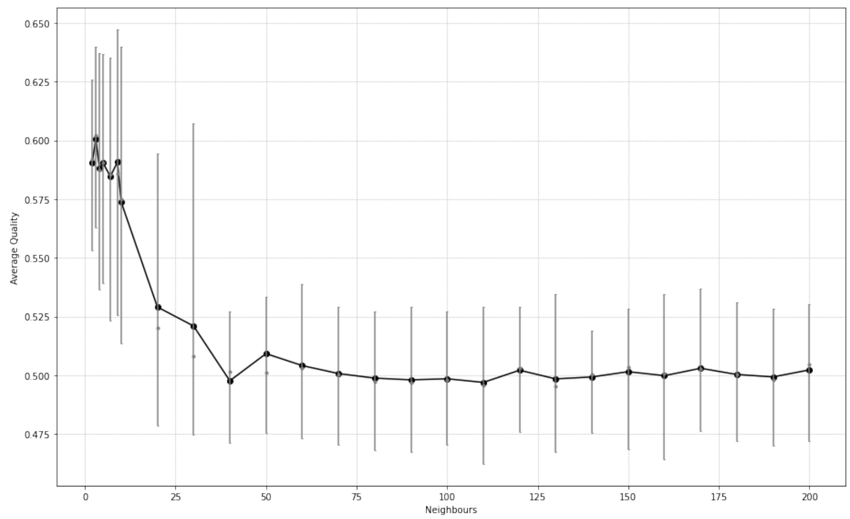
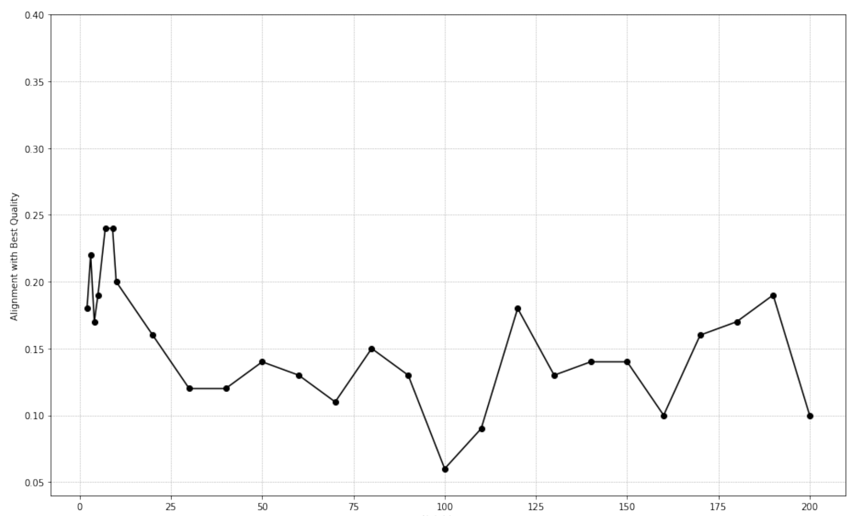


Figure B.1: Sigmoid Constrained to (0,1)

future use in swarm robotics and mimics the conclusion that we have arrived at, minimal connectivity yields better results.



(a) Average Quality



(b) Alignment with the best quality

Figure B.2: Performance Metrics for the Sigmoid-Based Threshold Function

C Appendix

C.1 Risk Register

Score	Likelihood	Impact
1	Very unlikely to happen	Almost no disruption
2	Unlikely to happen	Some disruption
3	Likely to happen	Significant disruption
4	Very likely to happen	Total stoppage of project

Table C.1: Risk assessment scale

Risk Description	Likelihood (L)	Impact (I)	Risk Score (L*I)	Mitigation
Unavailability of Required Robots	2	3	6	Adjust the research scope to further analysing some lesser effective models in simulations.
Failure to find effective Models	1	3	3	Examine why the current models have not been effective and determine what modifications can be made to the proposed method.
Failure to Run Simulations in Software	1	2	2	Ensure that the simulation software is compatible with the models and algorithms used. Verify the software's capabilities by running smaller tests before running full-scale simulations.
Inadequate Test Environment	2	1	2	Create a test environment that simulates the real world conditions as closely as possible. Conduct preliminary tests to identify and address any potential environmental issues.

Table C.2: Risk Register

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